

# Math 115

## Spring 2018

### Lecture 29

$$\begin{array}{l} ? a^2 + b^2 = c^2 ? \\ y = mx + b \quad ? \quad d = rt \end{array}$$

① Simplify :  $\frac{4x^2 - 9}{2x^2 - 5x - 3} = \frac{(2x+3)(2x-3)}{(2x+1)(x-3)}$

Non-reducible

② Find all excluded values:  $\frac{3x+5}{2x^2+5x-12}$

Deno. = 0, Solve

$$2x^2 + 5x - 12 = 0$$

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a=2 \quad b=5 \quad c=-12$$

$$b^2 - 4ac = 5^2 - 4(2)(-12)$$

$$= 25 + 96$$

$$= 121$$

E.V.:  
-4, 3/2

$$x = \frac{-5 \pm \sqrt{121}}{2(2)}$$

$$= \frac{-5 \pm 11}{4}$$

$$x = \frac{-5+11}{4}$$

$$x = 3/2$$

$$x = \frac{-5-11}{4}$$

$$x = -4$$

① Simplify:  $\frac{2x+1}{x^2+5x} \div \frac{6x^2-x-2}{x^3-25x}$

$$= \frac{2x+1}{x(x+5)} \cdot \frac{x(x^2-25)}{6x^2-x-2} = \frac{\cancel{2x+1}}{\cancel{x(x+5)}} \cdot \frac{\cancel{x(x+5)(x-5)}}{\cancel{(2x+1)(3x-2)}}$$

$$= \frac{x-5}{3x-2}$$

② Simplify:  $\frac{8}{x^2+6x} - \frac{4}{x^2+4x-12}$

$$= \frac{8(x-2)}{x(x+6)(x-2)} - \frac{4x}{(x+6)(x-2)x} = \frac{8(x-2) - 4x}{x(x+6)(x-2)}$$

$$= \frac{8x-16-4x}{x(x+6)(x-2)}$$

$$= \frac{4x-16}{x(x+6)(x-2)}$$

Simplify:  $\frac{\frac{5}{x} - 1}{\frac{5-x}{2x}}$

$$\text{LCD} = 2x$$

$$= \frac{\cancel{2x} \cdot \frac{5}{\cancel{x}} - 2x \cdot 1}{\cancel{2x} \cdot \frac{5-x}{\cancel{2x}}} = \frac{10 - 2x}{5-x} = \frac{2(5-x)}{\cancel{5-x}}$$

Solve:  $\frac{2}{x} - \frac{x}{x+3} = \frac{1}{x+3}$

$$= 2$$

LCM =  $x(x+3)$  E.V.: 0, -3

$$\cancel{x(x+3)} \cdot \frac{2}{\cancel{x}} - \cancel{x(x+3)} \cdot \frac{x}{\cancel{x+3}} = \cancel{x(x+3)} \cdot \frac{1}{\cancel{x+3}}$$

$$2(x+3) - x^2 = x$$

$$2x + 6 - x^2 - x = 0$$

$$-x^2 + x + 6 = 0$$

Multiply by -1 to make L.C. 1.

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

E.P.R.

$$x-3=0$$

$$x=3$$

$$x+2=0$$

$$x=-2$$

$$\{-2, 3\}$$

Simplify  $\frac{x - \frac{9}{x}}{1 + \frac{3}{x}}$

LCD =  $x$

$$\frac{x \cdot x - \cancel{x} \cdot \frac{9}{\cancel{x}}}{x \cdot 1 + \cancel{x} \cdot \frac{3}{\cancel{x}}} = \frac{x^2 - 9}{x + 3}$$

$$= \frac{x^2 - 9}{x + 3} = \frac{(x+3)(x-3)}{\cancel{x+3}} = x - 3$$

$x - 3$

Solve:  $\frac{3}{x-2} + \frac{1}{x-1} = \frac{7}{x^2 - 3x + 2}$

LCD:  $(x-2)(x-1)$  E.V.  $\neq 1$

$$3(x-1) + 1(x-2) = 7$$

$$3x - 3 + x - 2 = 7$$

$$4x - 5 = 7$$

$$4x = 12$$

$$x = 3$$

$\{3\}$

Simplify:  $\frac{4}{x^2 - 4x + 4} - \frac{3}{x - 2}$

$$= \frac{4}{(x-2)(x-2)} - \frac{3(x-2)}{(x-2)(x-2)} = \frac{4 - 3(x-2)}{(x-2)^2}$$

$$= \frac{4 - 3x + 6}{(x-2)^2} = \frac{10 - 3x}{(x-2)^2}$$

Simplify:  $\frac{x^{-2} - y^{-2}}{x^{-1} - y^{-1}}$

Hint:  $x^{-n} = \frac{1}{x^n}$

$$= \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} - \frac{1}{y}} = \frac{\cancel{x^2}y^2 \cdot \frac{1}{\cancel{x^2}} - \cancel{x^2}y^2 \cdot \frac{1}{\cancel{y^2}}}{\cancel{x^2}y^2 \cdot \frac{1}{\cancel{x}} - \cancel{x^2}y^2 \cdot \frac{1}{\cancel{y}}}$$

LCD =  $x^2y^2$

$$= \frac{y^2 - x^2}{xy^2 - x^2y} = \frac{(y+x)(y-x)}{xy(y-x)} = \frac{y+x}{xy}$$

Cross-multiply &amp; Solve

$$\frac{x}{4-x} = \frac{2}{x}$$

E.V. 4, 0

$$x \cdot x = 2(4-x)$$

$$x^2 = 8 - 2x$$

$$\rightarrow x^2 - 8 + 2x = 0$$

$$x^2 + 2x - 8 = 0$$

$$a=1 \quad b=2 \quad c=-8$$

$$b^2 - 4ac = 2^2 - 4(1)(-8)$$

$$= 36$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{36}}{2(1)} = \frac{-2 \pm 6}{2} \left\{ \begin{array}{l} \rightarrow x = \frac{-2+6}{2} = 2 \\ \rightarrow x = \frac{-2-6}{2} = -4 \end{array} \right\} \{-4, 2\}$$

Cross-multiply &amp; Solve

$$\frac{2x-3}{4} = \frac{4}{2x+3}$$

$$(2x-3)(2x+3) = 4 \cdot 4$$

$$4x^2 - 9 = 16$$

$$4x^2 - 9 - 16 = 0$$

$$4x^2 - 25 = 0$$

Solve by Factoring

$$(2x+5)(2x-5) = 0$$

$$2x+5=0$$

$$2x-5=0$$

$$x = -\frac{5}{2}$$

$$x = \frac{5}{2}$$

$$\left\{ \pm \frac{5}{2} \right\}$$

Solve by Q-Formula:

$$a=4 \quad b=0 \quad c=-25$$

$$b^2 - 4ac = 0^2 - 4(4)(-25) = 400$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{400}}{2(4)} = \frac{0 \pm 20}{8} = \pm \frac{5}{2}$$

$$\left\{ \pm \frac{5}{2} \right\}$$

Solve  $\frac{x}{x-3} + \frac{4}{x+3} = \frac{18}{x^2-9}$

LCD =  $(x-3)(x+3)$ , E.V.  $\pm 3$

$$x(x+3) + 4(x-3) = 18$$

$$x^2 + 3x + 4x - 12 - 18 = 0$$

$$x^2 + 7x - 30 = 0$$

① Solve by factoring:  $(x+10)(x-3) = 0$

$$x = -10 \quad x = 3$$

$$\{-10\}$$

② Solve by Q-Formula:

$$a=1, \quad b=7, \quad c=-30$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac = 7^2 - 4(1)(-30) = 169$$

$$x = \frac{-7 \pm \sqrt{169}}{2(1)}$$

$$x = \frac{-7 \pm 13}{2} \quad \rightarrow x = \frac{-7+13}{2} = \frac{6}{2} = 3 \quad \text{is an E.V.}$$

$$x = \frac{-7-13}{2} = \frac{-20}{2} = -10 \quad \{-10\}$$

Class Q7:

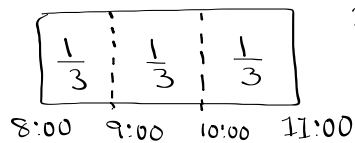
① Simplify:  $\frac{4x^2 - 25}{4x^2 + 20x + 25}$

② Solve by Q-formula:  $3x^2 - 5x + 2 = 0$

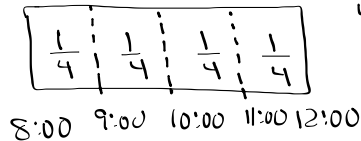
## work Problems

$$\begin{array}{ccc} \text{Amount of} & & \text{How long} \\ \text{work} & = & \text{You} \\ \text{completed} & & \text{work} \\ & & \underbrace{\hspace{2cm}} \\ & & \text{time} \end{array} \cdot \begin{array}{c} \text{How fast} \\ \text{You} \\ \text{work} \\ \underbrace{\hspace{2cm}} \\ \text{Rate} \end{array}$$

$$\begin{array}{c} \text{Amount of} \\ \text{work} \end{array} = t \cdot r$$



3 hrs to paint this wall  
Rate  $\frac{1}{3}$  /hr.



4 hrs to Paint the  
Same Wall  
Rate  $\frac{1}{4}$  /hr.

How long does it take to paint the wall  
working together?

$$\begin{array}{ccccc} \text{work} & & \text{work} & & \text{one} \\ \text{by} & + & \text{by} & = & \text{complete} \\ \text{You} & & \text{me} & & \text{work} \\ \text{Rate} & & \text{Rate} & & \\ \downarrow & & \downarrow & & \\ \frac{1}{3} \cdot t & + & \frac{1}{4} \cdot t & = & 1 \end{array}$$

$$\frac{t}{3} + \frac{t}{4} = 1$$

$$\text{LCD} = 12$$

$$4t + 3t = 12$$

$$7t = 12$$

$$t = \frac{12}{7}$$

$$t \approx 1.7 \text{ hrs}$$

Pipe A Can Fill up an empty Pool in 8 hrs alone.

Pipe B can do the same thing in 10 hrs alone.

How long does it take to fill up the pool if both pipes are used together?

$$\begin{array}{rcl}
 \text{Rate Pipe A} & + & \text{Rate Pipe B} = \text{complete work} \\
 \downarrow & & \downarrow \\
 \frac{1}{8} \cdot t & + & \frac{1}{10} \cdot t = 1
 \end{array}$$

$\frac{t}{8} + \frac{t}{10} = 1$   
 LCD = 40  
 $5t + 4t = 40$   
 $9t = 40$      $t = \frac{40}{9}$   
 $t \approx 4.4$  hrs

Faucet can fill up an empty sink in 6 minutes.

If the sink is full and drain is open, it takes 10 minutes to empty the sink.

Assume sink is empty, drain is left open. How long does it take to fill up the sink?

$$\begin{array}{rcl}
 \text{Faucet} & - & \text{Drain} = \text{complete work} \\
 \frac{1}{6} \cdot t & - & \frac{1}{10} \cdot t = 1
 \end{array}$$

LCD = 30  
 $5t - 3t = 30$   
 $2t = 30$   
 $t = 15$   
 15 minutes

John and Maria can paint a room together in 4 hrs.

John can paint alone in 7 hrs.

How long does it take Maria to paint the room alone?

work by John + work by Maria = complete work

$$\frac{1}{7} \cdot 4 + \frac{1}{x} \cdot 4 = 1 \quad \frac{4}{7} + \frac{4}{x} = 1$$

$$x = \frac{28}{3}$$

$$x \approx 9.3 \text{ hrs}$$

$$\text{LCD} = 7x$$

$$4x + 28 = 7x$$

$$28 = 3x$$

It takes John 9 hrs longer than Maria to do a certain job.

They can finish the job in 6 hrs working together. How long if they work alone?

Maria  $\rightarrow x$  hrs  $\rightarrow \frac{1}{x}$

John  $\rightarrow x+9$  hrs  $\rightarrow \frac{1}{x+9}$

work by Maria + work by John = complete work

$$\frac{1}{x} \cdot 6 + \frac{1}{x+9} \cdot 6 = 1 \quad \text{LCD: } x(x+9)$$

$$\frac{6}{x} + \frac{6}{x+9} = 1$$

$$6(x+9) + 6x = x(x+9)$$

$$6x + 54 + 6x = x^2 + 9x$$

$$12x + 54 = x^2 + 9x$$

$$x^2 + 9x - 12x - 54 = 0$$

$$x^2 - 3x - 54 = 0$$

$$(x-9)(x+6) = 0$$

$$x = 9$$

$$\cancel{x = -6}$$

Maria  $\rightarrow 9$  hrs

John  $\rightarrow 18$  hrs